Weighted Pluriregularity in $\mathbb{C}^n$

by

Muhammed Ali Alan

We will discuss the weighted Pluriregularity notions in $\mathbb{C}^n$. We will give answers to some problems posed by Sadullaev.

Shift operators on Hilbert harmonic function spaces

by

H. Turgay Kaptanoğlu

In an attempt to identify the harmonic Drury-Arveson space, we introduce and investigate large families of reproducing kernel Hilbert spaces of harmonic functions the unit ball of $\mathbb{R}^n$. Using zonal harmonics, we define and develop basic properties of shift operators and their adjoints in the harmonic setting. We obtain a dilation result for the shift operators that are row contractions. As a consequence, we show that the norm of one of our spaces is maximal among those spaces on which the shift operator is a row contraction.

This is an ongoing joint work with Daniel Alpay of Chapman University, Orange, CA.

Pluripotential theory associated to a convex body and Robin functions

by

Norman Levenberg

Given a convex body C in $(\mathbf{R}^+)^d$, one can associate a natural class of plurisubharmonic functions L_C on $\mathbf{C}^d$: those that grow like the logarithmic indicator function

$$H_C(z_1, ..., z_d) := \phi_C(\log|z_1|, ..., \log|z_d|), \ z_j \in \mathbf{C}$$

where $\phi_p(x_1, ..., x_d) := \max\{a_1x_1 + \cdots + a_dx_d : (a_1, ..., a_d) \in C\}, \ x_j \in \mathbf{R}$ is the usual support function of C. These generalizations of Lelong classes in standard pluripotential theory arise in the theory of random sparse polynomials and in problems involving polynomial approximation. We discuss results in the general theory (mostly due to Bayraktar, Bloom, Bos, L., Ma'u, Piazzon) and at the end we outline a particularly difficult object to define: a $C-$Robin function associated to a function $u \in L_C$ (joint work in progress with S. Ma'u).

Eigensystems of non-self-adjoint operators

by

Boris S. Mityagin

The lecture focuses on the geometry of systems of eigenvectors of non-self-adjoint operators. The main examples come from analysis of two families of ODE operators:

(a) Hill-Schrödinger operators

$$Ly = -y'' + v(x)y, \quad 0 \le x \le \pi, \quad v(x + \pi) = v(x);$$

(b) the harmonic oscillator and its perturbations

$$My = -y'' + x^2y + w(x)y, \quad -\infty < x < \infty.$$

Topics to be Discussed

- Instability zones (spectral gaps) of an operator (a).

  Special attention is given to their asymptotics in the case of a two-term potential

  $$v(x) = a\cos(2x) + b\cos(4x).$$

- Spectral decompositions of Hill operators with trigonometric polynomial potentials.

- Root Systems of Perturbations of Harmonic-Oscillator-Type Operator

- Differential Operators admitting various rates of spectral projection growth

The presentation is based on, but not limited to, the work of the lecturer and his coauthors P. Djakov, J. Adduci, P. Siegl, and J. Viola.

Interpolation of weighted extremal functions

by

Alexander Rashkovskii

We propose an approach to interpolation of compact subsets of $\mathbb{C}^n$ by means of plurisubharmonic geodesics between weighted relative extremal functions of the given sets. In particular, we establish convexity properties of the capacities of the interpolating sets that are stronger than those given by the Brunn-Minkowski inequalities.

Oscillatory integrals and Weierstrass polynomials

by

Azimbay Sadullaev

The well-known Weierstrass theorem states that if $f(z,w)$ is holomorphic at a point $(z^0,w^0) \in \mathbb{C}^n_z \times \mathbb{C}_w$ and $f(z^0,w^0) = 0$, but $f(z^0,w) \not\equiv 0$, then in some neighborhood $U = V \times W$of this point f is represented as

$$f(z,w) = \left[\left(w-w^0\right)^m + c_{m-1}(z)\left(w-w^0\right)^{m-1} + \ldots + c_0(z)\right]\varphi(z,w), \quad (1)$$

where $c_k(z)$ are holomorphic in V and $\varphi(z,w)$ is holomorphic in U, $\varphi(z,w) \neq 0$, $(z,w) \in U$.

In recent years, the Weierstrass representation (1) has found a number of applications in the theory of oscillatory integrals. Using a version of Weierstrass representation the first author (see [1]) obtained a solution of famous Sogge - Stein problem (see [2]). He obtained also close to a sharp bound for maximal operators associated to analytic hypersurfaces.

In the obtained results the phase function is an analytic function at a fixed critical point without requiring the condition $f(z^0,w) \not\equiv 0$. It is natural to expect the validity of Weierstrass theorem without requiring a condition $f(z^0,w) \not\equiv 0$, in form

$$f(z,w) = \left[c_m(z)\left(w-w^0\right)^m + c_{m-1}(z)\left(w-w^0\right)^{m-1} + \ldots + c_0(z)\right]\varphi(z,w).(2)$$

Such kind of results may be useful to studying of the oscillatory integrals and in estimates for maximal operators on a Lebesgue spaces. However, the well-known Osgood counterexample [3], p.90 (see also [4], p. 68) shows that when $n > 1$ it is not always possible.

In the talk we will discuss, that there is a global option (see [5],[6]), also a global multidimensional (in w) analogue of (2) is true without requiring the condution $f(z^0,w) \not\equiv 0$. In addition, for an arbitrary germ of a holomorphic function, we will prove one representation, that is useful in the study of oscilatory integrals.

References

[1] I.A.Ikromov, Damped oscillatory integrals and Maximal Operators, Math. Notes, V.78(2005), no. 5-6, 773-790.

[2] C.D. Sogge and E.M. Stein, Averages of functions over hypersurfaces in $\mathbb{R}^n$, Invent. Math., V. 82(1985), no. 3, 543-556.

[3] W. Osgood, Lehrbuch der Funktionentheorie, Bd.II, Teubner, Leipzig, 1929.

[4] B. A. Fuks, Introduction to the theory of analytic functions of several complex variables, (in Russian), Moscow, 1962.

[5] A. Sadullaev, Criteria algebraicity of analytic sets, Functional analysis and its application, V.6: 1, 1972, 85-86. (in Russian).

[6] A. Sadullaev, Criteria algebraicity of analytic sets, Institute of Physics named after L.V. Kirensky, Krosnoyarsk, 1976, pp. 107-122. (in Russian).

Compactness and schatten class Hankel operators

by

Sönmez Şahutoğlu

We discuss compactness and Schatten class properties of Hankel operators defined on Bergman spaces on domains in $\mathbb{C}^n$. This is joint work with Nihat Gökhan Göğüş

Surjectivity of Euler-type differential operators on spaces of distributions

by

DIETMAR VOGT

Euler partial differential operators are operators of the form $P(\theta)$ where P is a polynomial in d variables and $\theta_j = x_j\partial/\partial x_j$ the Euler derivative. They are partial differential operators with variable coefficients which are singular at the coordinate hyperplanes. In contrast to partial differential operators with constant coefficients they admit distributional zero solutions with compact support, located at the singular locus.

It was shown by Domański and Langenbruch that on $C^\infty(\mathbb{R}^d)$ they are surjective onto the annihilator of these zero solutions, in particular they are, in general, not surjective but have closed range.

We show that, surprisingly, they are surjective on the space of temperate distributions on $\mathbb{R}^d$. This holds also for the space of finite order distributions on $\mathbb{R}^d$ and of Schwartz distributions on $\mathbb{R}$. On $\mathscr{D}'(\mathbb{R}^d)$ they have always dense range, but for $d \geq 3$ they are, in general, not surjective. Surjectivity on $\mathscr{D}'(\mathbb{R}^2)$ is unknown.

A remark on a paper of P. B. Djakov and M. S. Ramanujan

by

MURAT YURDAKUL

Let ℓ be a Banach sequence space with a monotone norm in which the canonical system (e_n) is an unconditional basis. We show that if there exists a continuous linear unbounded operator between ℓ-Köthe spaces, then there exists a continuous unbounded quasi-diagonal operator between them. Using this result, we study in terms of corresponding Köthe matrices when every continuous linear operator between ℓ-Köthe spaces is bounded. As an application, we observe that the existence of an unbounded operator between ℓ-Köthe spaces, under a splitting condition, causes the existence of a common basic subspace.